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A New Approach for Design of Model Matching Controllers for Time Delay Systems by Using GA Technique

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ABSTRACT
Modeling of physical systems usually results in complex high order dynamic representation. The simulation and design of controller for higher order system is a difficult problem. Normally the cost and complexity of the controller increases with the system order. Hence it is desirable to approximate these models to reduced order model such that these lower order models preserves all salient features of higher order model. Lower order models simplify the understanding of the original higher order system. Modern controller design methods such as Model Matching Technique, LQG produce controllers of order at least equal to that of the plant, usually higher order. These control laws are may be too complex with regards to practical implementation and simpler designs are then sought. For this purpose, one can either reduce the order the plant model prior to controller design, or reduce the controller in the final stage, or both. In the present work, a controller is designed such that the closed loop system which includes a delay response(s) matches with those of the chosen model with same time delay as close as possible. Based on desired model, a controller(of higher order) is designed using model matching method and is approximated to a lower order one using Approximate Generalized Time Moments (AGTM) / Approximate Generalized Markov Moments (AGMM) matching technique and Optimal Pade Approximation technique. Genetic Algorithm (GA) optimization technique is used to obtain the expansion points one which yields similar response as that of model, minimizing the error between the response of the model and that of designed closed loop system.
***Keywords* -** Approximate Generalized Time Moments (AGTM), Genetic Algorithm (GA)

I. INTRODUCTION

During the last decade, it has been shown that wide classes of control systems such as chemical process, national economy, traffic networks, steam quality control, cold rolling mill, etc., may be modelled in terms of time-delay systems.

Time delays which occur between the inputs and outputs of physical systems are often found in industrial systems, in particular process control, economical and biological systems. Typical sources of time delays are associated with transportation and measurement lags, analysis times for sensor measurement, computation and communication lags. The presence of time delays in a system may make the design of feedback controllers for a system more demanding, since time delay tends to destabilize a system. Time delays always reduce the stability of systems. The control action cannot be realized immediately because of the time delay. This can lead to instability of a system. The effect of the time delay on the system dynamics, however, depends on the delay and the system characteristics. So, time delay systems present a wide range of challenges in implementing controllers for them.

Problem Definition

To design a controller such that the closed loop system response(s) matches with those of the chosen model as close as possible.

- A desired model should be developed for the specified performance measures. Based on desired model, a controller(of higher order) will be designed using Model Matching method [11] and will be further approximated to a lower order one using Approximate Generalized Time Moments (AGTM) / Approximate
- Generalized Markov Moments (AGMM) matching technique[1] and Optimal Pade Approximation technique[16].
- Any optimisation technique can be used to obtain the better expansion point i.e, a better response as that of model.

Methodology

- A frequency domain method, called polynomial approximation is used. Pade approximation is commonly use for model reduction. The main drawback of Pade approximation technique [5] is that stability of the resultant order model is not guaranteed.

- One of the methods proposed here, Polynomial approximation methods are based on Time Matching moment [2 4] and Markov parameters matches between original and reduced order model.

MODEL MATCHING APPROACH TO CONTROLLER DESIG

- The higher order controllers are found to be fragile which may even lead to instability for very small changes in the controller coefficient.
- The present work follows the first approach by designing a high order controller for the plant with specific performance.

There are two approaches for controller design,

- Exact Model Matching (EMM)
- Approximate model matching (AMM) EXACT

AGTMmethodemismatchingthefrequencyresponseofactualandapproximatedmodelsatdifferentpointsinthesplane.Thesepointsareexpansionpoints(nonzerorealvalues).Thenumberofexpansionpointsissameasthenumberofunknownsinthe Eqn.Equatetheresponseofactualandapproximatetransferfunctionatexpansionpoints.Substitutetheexpansionpointsin Eqn.andgettheequationswithunknownparameters.Here,inthiscaseunknownisintheexponentsoequationsarenonlinear.Solvetheseequationswithaninitialvector_x0‘.

$$x_0 = [b_{m,0}\, b_{m-1,0} \ldots b_{1,0}\, b_{0,0}\, a_{n,0}\, a_{n-1,0} \ldots a_{1,0}\, a_{0,0}\, \tau_{d,0}]$$

Thesolutionofequationsareunknowncoefficientsofapproximatemodeltransferfunctionandtimedelay.Byusingthissolutionformtheapproximatemodelwithtimedelay.Getthestepresponsedataofthisapproximatedmodel.Thematchingeffectivenessoftheapproximatemodelisbasedontheperformanceindexvalue

$$J = \int_0^{\infty} (y_a(t) - y_m(t))^2 dt$$

where y(t)is actual MTDS step response and y(t) is Approximated model step response. Search for minimum‘ by varying expansion points, initial vector or both and select the corresponding model as the best approximated model.

II. TIME DELAY SYSTEMS

Time delays often arise in control systems, both from delays in the process itself and from delays in the processing of sensed signals. Process industries often have processes with time delays introduced due to the finite time it takes for material to flow through pipes. In measuring altitude of a spacecraft, there is a significant time delay before the sensed signal arrives back on Earth. A recent example of it is interplanetary telecommunication through Mars rover. In modern digital control systems, time delay can arise from sampling, due to cycle time of the computer and the fact that data is processed at discrete intervals. Thus, time delay could be due to heat and mass transfer in chemical industries, heavy computations and hardware restrictions in computational systems, high inertia in systems with heavy machinery and communications lag in space craft and remote operation. Chemical processing systems, transportation systems, communication systems and power systems are typical examples that exhibit time-delays. The effect of the time delay on the system dynamics, however, depends on the delay and the system characteristics.

Time delays fall into two main categories:

1. Fixed time delay
2. Time-varying delay

Fixed time delay

Delays, which remain constant with time are called fixed time delays. Figure 2.1 shows a fixed time delay of T sec. The Laplace Transform of the system output is

y(s) =e-stu(s)

Where

$$e^{-s\tau} = \frac{1 - s\frac{\tau}{2} + s(\frac{\tau}{2})^2 - \mathrm{L}}{1 + s\frac{\tau}{2} + s(\frac{\tau}{2})^2 + \mathrm{L}}$$

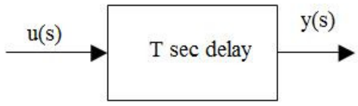


Fig.1. Fixed delay block

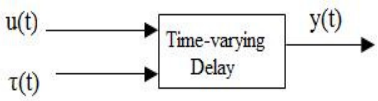
In the time domain, we can write it as

y(t)=u(t−T)

A substantial work has been done in the past on the approximation of the constant delay. Many equivalent frequency domain [15] Transfer Functions have been proposed to describe constant time delay systems. The methods that are employed are closely related to the

Time-varying delay

Delays, which are functions of time, are called time-varying delays. Linear systems with time-varying delays may be represented as



$$x\&(t) = (A + \Delta A(t))x(t) + (A\,d1 + \Delta A\,d1\,(t))x(t - d1\,(t)) + (B + \Delta B(t))x(t) + (Bd\,1(t))x(t - d1\,(t))$$

$$y(t) = Cx(t)$$

and all the matrices have appropriate dimensions

Time-varying delay systems show significantly different characteristics from that of fixed time delay systems. Satisfactory modeling of time-varying delay is important for the synthesis of effective control systems for such systems.

III. OVERVIEW OF MODEL ORDER REDUCTION AND CONTROLLER DESIGN METHODS

A first type of classification can be given by referring to the domain where the high-order and low-order models have to be represented: either frequency or time. This is quite general, and does not refer to any particular system structures (i.e. MIMO-SISO, symmetric, asymmetric). From an operative viewpoint, another type of classification is suggested by Skelton, who indicates three categories of model reduction procedures:

1. Methods based on polynomial approximations (usually suitable in the frequency domain) [12]
2. Component truncation procedures based on state-space transformations
3. Parametric optimization techniques

Importance of model order reduction

The model reduction philosophy is a natural procedure in engineering practice. The main reasons for obtaining low-order models can be grouped as follows:

1) To have low-order models so as to simplify the understanding of a system
2) To reduce computational efforts in simulation problems
3) To decrease computational efforts and so make the design of the controller numerically more efficient.
4) To obtain simpler control laws

MODEL MATCHING APPROACHES TO CONTROLLER DESIGN

Modern robust controller techniques like H∞, LQR and Linear Quadratic Gaussian (LQG) methods lead to complex controllers, the orders of which may often be equal to or more than that of the plant itself. These resultant high order controllers are found to be highly sensitive to quantization error and are often found to be fragile which may even lead to instability for very small changes in the controller coefficients. The present work follows the first approach by designing a high order controller for the plant with specified performance. This will give the desired response and desired model. There are two approaches for controller design, exact model matching (EMM) and approximate model matching (AMM)procedures.

Exact Model Matching (EMM) problem

In the exact model matching problem, it is desired to find the unknown parameters of the controller C(s) such that the closed loop transfer function T(s) exactly matches a general specification transfer function M(s).

$$\varsigma(s) = a(s)\, k(s)+c(s)\, h(s) = \hat{\varsigma}(s)\, \hat{q}(s)$$

A generalized algorithm has prepared for this method. But it has the drawbacks like pole-zero cancellation of the polynomial $\hat{q}(s)$ in closed loop transfer function corresponds to a lack of closed loop system controllability, orders of q(s), h(s) and k(s) are fixed, Diophantine equation uses exact model matching, choice of ς(s) is restricted, etc.

Approximate Model Matching (AMM) problem

The above difficulties of EMM can be effectively removed by using the concept of approximate model matching (AMM) procedures. So here an approximate model matching method will be used. By AMM technique, it is possible to design a compensator of fixed order and structure to satisfy the desired specifications embodied by a general reference transfer function M(s), having no restrictions on its order. So here an approximate model matching method has used.

Controller design by pade type approximation technique

Pade type approximation techniques have been widely used in the area of reduced order modeling. In the area of reduced order modeling, the objective is to find a reduced model R(s) that approximates a stable high order system G(s). The Pade approximation technique matches two sets of parameters called the time moments Ti and Markov parameters Mi, of G(s) with the corresponding parameters of R(s).

Time Moments

An irreducible rational function G(s) can be expanded as:

$$G(s) = \int_0^{\infty} (g(t)e^{-st})\, dt$$

$$G(s) = \int_0^{\infty} (g(t)e^{-st})\, dt$$

$$- s\int_0^{\infty} tg(t)dt + s^2 \int_0^{\infty} \frac{t^2}{2!}\, g(t)dt$$

Where g(t) is the impulse response of G(s).
Expanding G(s) into its power series expansion

$$G(s) = c_0 + c_1 s + c_2 s^2 + \cdots = \sum_{i=0}^{\infty} c_i\, s^i$$

the expression for the i^{th} derivative of G(s) evaluated at s=0 as

$$\left\{\frac{d^i}{ds^i} G(s)\right\}\Bigg|_{s=0} = (-1)^i \int_0^{\infty} t^i g(t)dt \triangleq (-1)^i T_i$$

Where is defined as the i^{th} Time Moment (TM) of G(s).it may be shown that

$$c_i = \frac{1}{i!}\frac{d^i}{ds^i} G(s)\,\Bigg|_{s=0} = \frac{(-1)^i}{i!} T_i$$

C_i may be thus called the proportional Time Moment of G(s).for the state space triple (A,B,C). it may be shown that :

$$T_i = (-1)^i \, CA^{-(i+1)}B; \qquad i = 1,2, \dots \infty$$

Markov Parameters

Expansion of a strictly proper rational function G(s) into a power series expansion about infinity (s=) yields:

$$G(s) = \sum_{i=0}^{\infty} \frac{1}{s^{i+1}} M_i = \sum_{i=0}^{\infty} CA^i B s^{-(i+1)}$$

Then the coefficients of the series are given as:

$$M_i = CA^i B; \qquad i = 0,1,2,3\dots\infty$$

These coefficients are called Markov Parameter (MP) of the dynamic system

$$\text{If } g(t) = \mathcal{L}^{-1}(G(s)) = Ce^{At}B,$$

be the impulse response of the system, then:

$$CA^iB = M_{i+1} = \frac{d^i}{ds^i} g(t)|_{t=0}; \qquad i = 0,1,2, \dots \infty$$

It has been reported in the literature on reduced order modeling that matching initial few time moments ! of G(s) and R(s) ensures good matching in the low frequency response (steady-state) while matching initial few Markov parameters of the respective systems ensures good matching in the high frequency zone (transient response). For controller design by using Pade approximation techniques [4], [7], one may match initial few TM and MP.

Mathematical Preliminaries for AGTM matching

Consider a real function with derivatives, i=1,2,… in some region around the point x0. Let the values of F(x) be given for the real numbers x0, x1, x2,…, xn of the variables xi; where $x_i = x_0 + h$; i ∈[1,n] and $h > 0$. These latter numbers x; are supposed to be all different. Using the notation of the calculus of divided differences, we may define **f[x] ≜ f(x_0)** and following divided difference of arguments 2 to (n+1):$f[x_0,x_1] \triangleq (f[x_1]-f[x_0])/(x_0-x_1)$
$f[x_0,x_1,x_2] \triangleq (f[x_0,x_1]-f[x_2,x_1])/(x_0-x_2)$
$f[x_0,x_1,x_2\text{------}x_n] \triangleq (f[x_0,x_1,x_2\text{------}x_{k-1}]-f[x_1,x_2\text{------}x_k]/(x_0-x_k)$;k∈[2,n]

Suppose that in the interval (a, b) bounded by the greatest and least of x0, x1, x2,…,xn the function of the variable x and its first (n-1) derivatives are finite and continuous and that exists. It may then be shown that

$$f[x_0, x_1, \dots x_n] = h^n \sum_{i=0}^{n} \frac{(-i)^{n-1}}{i!(n-1)!} f(x_1) = \frac{1}{\eta} f^{(n)}(\eta)$$

Where n lies in the interval X0≤n<(X0+nh)

Now, let ψ(x) be a second real function with finite and continuous derivatives around the point x = x0 such that

ψ(X0) = f(Xi), i=0,1,2,…n (4.13)

(X0 + n*h*).if the parameter h takes a very small non-negative value; then

f(i)(X0) ≅ Ψ(i)(X0), i ∈[0,n] (4.14)

Thus, for a suitably small value of the parameter h for a given another real valued function Ψ(x) may always be constructed using (4.13) so that the approximate relation in (4.14) are satisfied.

Approximate Generalized Time Moments

Let K(s) be the high order controller transfer function obtained by the Synthesis equation. The time moments ! ; T i=0,1,2, … , ∞ of K(s) are defined as

$$T_i = (-1)^i \frac{d^i}{ds^i} K(s)\Big|_{s=0} \qquad (4.15)$$

are proportional to where:

$$K(s) = \sum_{i=0}^{\infty} c_i \, s^i \qquad (4.16)$$

In the Classical Pade Approximate (CPA) technique, the reduced order transfer function equivalent of K(s) is chosen as

$$C(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \dots\dots\dots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \dots\dots\dots + a_1 s + a_0}; \; n>m \qquad (4.17)$$

Where n and m are the chosen orders of the denominator and numerator respectively. The denominator is assumed to be a manic polynomial .ai,i =0,1,2 ………(n − 1) and bi,i " = 0,1,2 … . , m are the unknown parameter which are to be determined. C(s) is chosen such that its power series expansion coefficients η i

$$C(s) = \sum_{i=0}^{\infty} ns^i \qquad (4.18)$$

Coincide with the corresponding coefficients of K(s), i=0,1,2,…,(nc − 1), where is the number of expansion points. has to be chosen depending on the number of unknown parameters of the controller as well as on the type of the expansion points (real or complex) .

From (4.15), and (4.16) and (4.18), the CPA technique is mathematically equivalent to:

$$T_i \left\{ \frac{d^i}{ds^i}(K(s)) \right\}\Big|_{s=0} = \left\{ \frac{d^i}{ds^i}(C(s)) \right\}\Big|_{s=0} \; ; \; i \in [1, n_e] \qquad (4.19)$$

Using the results of mathematical preliminaries given in Section 4.4.3, the exact relations in (4.19) may be replaced by the approximate ones as in (4.13) and (4.14). This finally gives, using (4.19), (4.13) and (4.14)

$$K(\delta_i) = C(\delta_i), \qquad i \in [1, n_e] \qquad (4.20)$$

Where are suitable non-zero general numbers (frequency) which are termed as expansion points and is the number of expansion points taken .

Let be distinct values of such that $t_1, t_2, t_3 \ldots\ldots\ldots t_n$ is equivalent to i ϵ [1,ne] t_i are then defined.

Approximate Generalized Time Moments

(AGTM), generalized because the relations in(4.20) are similar to the power expansions of K(s), C(s)about a non-zero general points and approximate because the exact differential operations are replaced by the divided difference approximations, details of which are described in following section .

Selection of Expansion Points

In the area of reduced order modeling, the classical Pade approximation technique makes use of power series expansions of a rational function G (s) about s=0 or s=∞ leading respectively to the time moments or the Markov parameters. To alleviate the occasional instability problem encountered by Pade approximants, several authors have suggested expansion about s=a, where 'a' is a nonzero, non negative real number (frequency). Based on the s-plane distribution of the poles and zeros of G(s), Pal.J and Ray.L.M [7] have proposed several heuristic criteria for choosing a feasible value of 'a'. It has been shown by Lucas.T.N that expansions about negative or complex points in the s-plane may lead to better or 'optimal' approximations.

In this project, a method is proposed which finds the 'optimal' points of expansion in the s-plane that finally leads to an approximation which is best in the sense of minimizing a user defined performance index .The expansion points can be a positive or negative real number or a complex point chosen from any of In this project, a method is proposed which finds the 'optimal' points of expansion in the s- plane that finally leads to an approximation which is best in the sense of minimizing a user defined performance index .The expansion points can be a positive or negative real number or a complex point chosen from any of the four quadrants of the s-plane. Care must be exercised that in case of choosing complex points; these should not be in conjugate pairs. In the controller design scenario, the choice of the expansion points, are governed by the stability and performance of the closed loop system. The controller C (s) has to be designed such that the closed loop system responses satisfy the desired specifications, while guaranteeing closed loop stability as well. No theory is yet available to determine or search the expansion points, such that poles of the resulting closed loop system can be guaranteed to be in the left half of the s-plane. The number of expansion points, depends upon the number of unknown parameters. The problem of choosing the best expansion points in order to yield a stable response as that of the model, can be chosen as a constrained optimization problem for the controller design, which is solved by Genetic Algorithm (GA) .

IV. CONTROLLER DESIGN BASED ON OPTIMAL PADE APPROXIMATION METHOD

A frequency domain method, called Optimal Pade Approximation(OPA) is commonly use for model reduction[16].The method was extended to fractional order controller design procedure by incorporating certain modifications and including the point of expansion at s= ∞.This method completely general for choice of expansion points, with little additional computational effort.

Lucas [16] has shown that for some systems the optimal reduced order models may require expansion points which are complex numbers and has suggested a method for generalizing the Optimal Pade Approximation (OPA) approach[4] which takes this into account, without requiring the use of any complex arithmetic. Reduced order models are derived by solving a set of linear pade equations[7] that allows a mixture of real, multiple or complex expansion points to be used and requires no complex arithmetic. The central idea of the Lucas technique[16] is to transform the rational approximation. This results in the method making use of elementary matrix operations and being computationally very efficient. The present work has modified and improved the Lukas method by relaxing the above mentioned restrictions and also extended the method to the controller design procedure. The higher and lower order transfer functions can now be proper or improper functions having no restrictions the relative order of the numerator and the denominator of either higher or lower order transfer functions.

Optimal Pade Approximation (OPA) method for controller design[5]

The main objective of model matching[11] is to design controllers in such a way that the step response of reference model should match or (as close as possible i.e. minimum error criteria) the step response of closed loop controlled plant.

Let K(s) be the higher order proper or improper controller transfer function obtained by model matching. K(s) can be represented in the general form as,

$$K(s)=\frac{b_{n-1}s^{n-1}+b_{n-2}s^{n-2}+\dots\dots+b_1 s+b_0}{a_n s^n+a_{n-1}s^{n-1}+\dots\dots+a_1 s+a_0} \quad (5.1)$$

Where $N_K(s)$ and $D_K(s)$ are respectively the numerator and denominator only nominal of K(s). b_i,i=0,1,2,.....q and a_i,i=0,1,2.....p are all known real numbers.

Implementation of the zero augmentation process [2] in K(s) extends the Lucas method and makes it more generalized[16,18].

$$K(s)=\frac{N_k(s)}{D_k(s)}=\frac{b_q s^q+b_{q-1}s^{q-1}+\dots\dots+b_1 s+b_0}{a_p s^p+a_{p-1}s^{p-1}+\dots\dots+a_1 s+a_0} \quad (5.1)$$

The zeros padded up as higher order coefficients do not add any value to the polynomial but when they enter as
matrix elements, this zeros attain lot of significance.
K(s) can be reduced to C(s) of chosen structure by
Optimal Pade Approximation (OPA) Method

$$C(s)=\frac{N_c(s)}{D_c(s)}=\frac{d_{m-1}s^{m-1}+\dots\dots+d_1 s+d_0}{s^m+e_{m-1}s^{m-1}+\dots\dots+e_1 s+e_0} \quad (5.3)$$

Where $N_C(s)$ and $D_C(s)$ are respectively the numerator and denominator polynomials of C(s). d_i,i=0,1,2,.....m and e_i,i=0,1,2.....m are unknown real numbers which are to be determined[6].

The proposed Optimal Pade Approximation (OPA) algorithm for a chosen set of expansion points s=S_i i=0, 1,2,..... (2m) is as followed

For model matching [15], the condition is

$$C(s)=K(s)$$

$$\frac{N_C(s)}{D_C(s)}=\frac{N_K(s)}{D_K(s)}$$

On cross multiplication, becomes

$$\boldsymbol{D_K(s)\,N_C(s)=N_K(s)\,D_C(s)} \quad (5.4)$$

L.H.S & R.H.S can be termed as P(s) & Q(s) respectively. Thus, expression for P(s) & Q(s) can be written as

$$P(s)=D_K(s)N_C(s)$$

$$P(s)=(a_n s^n+\dots\dots+a_1 s+a_0)\,(d_{m-1}s^{m-1}+\dots\dots+d_1 s+d_0) \quad (5.5)$$

$$Q(s)=N_K(s)D_C(s) \quad (5.6)$$

The degree of the polynomial P(s) & Q(s) is $(n+m-1)$

Equation (5.6) can be rewritten as

P(s) = Q(s) at some expansion points (5.7)

From the expansion points s=S_i where i=1,2,3,… where k is the number of unknown parameters of C(s) in (5.5)

Form a polynomial h(s) whose roots are the expansion points=S_i, as given below

$$h(s)=(s-S_1)(s-S_2)\dots\dots(s-S_k)$$

The central concept of Lucas method lies in the following statement:

"By the remainder theorem, dividing a general polynomial T(s) of degree r by another polynomial H(s), of degree v, (v<r), from highest powers, gives a reminder polynomial of degree (v-1) which is the Taylor approximation of T(s) about the „v" roots of H(S)". [15].Mathematical representation of reminder theorem is

$$T(s)\big|_{\text{roots of H(s)}}\approx R(s)\big|_{\text{roots of H(s)}}$$

This concept is applied to P(s) & Q(s) of (5.7 & 5.8 respectively) to obtain their Taylor approximant about the expansion

$$\frac{P(s)}{h(s)}=\frac{Q(s)}{h(s)}$$

Reminder theorem:-

$$P(s)=h(s)q(s)+R_p(s)$$

$$P(s)\big|_{\text{roots of h(s)}}=R_p(s)\big|_{\text{roots of h(s)}}$$

$$Q(s)=h(s)q(s)+R_q(s)$$

$$Q(s)\big|_{\text{roots of h(s)}}=R_q(s)\big|_{\text{roots of h(s)}}$$

From (5.9) $P(s)=Q(s)$ at expansion points then $R_p(s)\approx R_q(s)$ at roots of h(s)

V. SIMULATION RESULTS

- A second order model chosen is

$$M(s)=\frac{w_n^2}{s^2+2\xi w_n s+w_n^2}\,e^{-T_{dm}s}$$

Rise time (Tr) = 0.28 sec
Settling time (Ts) = 0.62 sec
Damping ratio ( ) = 0.9
Time delay(T_m) = 0.2 sec

A second order unstable plant G(s) with time delay (T_p) chosen is,

$$G(s)=\frac{3}{s^2+s}\,e^{-0.2s}$$

- The design of a controller is such a way that the step response of the model chosen should match (or as close as possible i.e. minimum error criteria) the step response of closed loop plant. Controller K(s) is designed using Model Matching Technique

$$K(s) = \frac{M}{(1 - M)G}$$

The obtained controller K(s) is of higher order. So, it is reduced to lower order structure using the following order reduction techniques.

1. AGTM/AGMP method [7]
2. Optimal Pade approximation method [15]

AGTM/AGMP Method [7]
Higher order controller K(s) is reduced to different lower order structure

Design of PID controller
General structure of PID controller is,

$$C(s) = \frac{K_D s^2 + K_p s + K_I}{s}$$

Where, K_P, K_I and K_D are the controller parameters.
For expansion points, s= [-0.8 0.2 -0.1],
PID controller.

$$C(s) = \frac{1.766S^2 + 1.742S + 0.0786}{S}$$

- Where, K_D=1.766;Kp=1.742;K_I=0.0786
- System is stable.
- Performance measure, J=30.3

Design of PI controller
General form of the PI controller is

$$C(s) = \frac{K_p s + K_I}{s}$$

For expansion points,s= [-0.1, -0.2]
- PI controller

$$C(s) = \frac{0.062s + 0.0127}{S}$$

Where, Kp=0.62,K_I =0.0127
- System is stable
- Performance measure, J=75.01

Design of practical PID controller
General form of the PPID controller is

$$C(s) = \frac{(K_p + K_D)s + (K_I + \lambda K_p)s + \lambda K}{s^2 + \lambda s}$$

As general PID controller is an improper transfer function, an insignificant pole is added to the controller transfer function to make it as proper. For expansion points, s= [0.05, 0.01, 0.0001], λ= 1000
- Practical PID controller
- System is stable

$$C(s) = \frac{533.4s^2 + 0.877s - 1.32e^{-9}}{s^2 + \lambda s}$$

Performance measure, J=4.9

Design of Lead/lag controller
General form of the Lead/Lag controller is,

$$C(s) = \frac{K(s+a)}{(s+b)}$$

For expansion points, s= [0.1, 0.2, 0.3]
- Lead/Lag controller,

$$C(s) = \frac{1.62s + 1.81}{s + 1.46}$$

- System is stable.
- Performance measure, J=48.8

Comparison of different controller performances using AGTM/AGMP method for randomly chosen expansion points:

Table 6.1 Comparison of different controller performance using AGTM/AGMP method for randomly chosen expansion points:

Type of Controller	Expansion Point	Parameters			Stability	Performance measure
		Kd	Kp	Ki		
PID	[0.1 0.2 0.3]	0.4	0.9038	-0.0014	0	-
	[0.1 1 0.5]	0.206	1.0073	-0.009	0	-
	[0.01 0.02 0.03]	0.5	-0.001	0.8775	1	38.78
	[-0.8 0.2 -0.1]	1.766	1.7425	0.0746	1	30.3
PI	[0.1 0.2]		-0.009	1.0239	0	-
	[0.01 1 0.02]		-0.002	0.894	1	95.4005
	[-0.01 -0.02]		-0.001	0.8605	1	91.74
	[-0.1 -0.2]		0.692	0.0127	1	75.01
PPID (I=100)	[1 2 3]	-191.1	1.83	-0.43	0	-
	[0.1 0.2 0.3]	400.5	0.9038	-0.0001	0	-
	[0.05 0.01 0.000	533.4	0.877	-1.32E-09	1	4.906
	[0.001 -0.1 -1]	909.2	0.9101	-3.30E-05	1	0.317
Lead/ Lag		K	a	b		
	[0.1 0.2 0.3]	1.67	1.81	1.46	1	48.8
	[0.01 0.02 0.03]	2.003	1.97	1.75	1	41.0379
	[-0.1 -0.2 0.01]	2.2159	2.09	1.94	1	37.027
	[0.5 -2 -4]	5.515	5.1941	3.9637	1	9.549

Figure 6.1: Step response of closed loop system with PID controller for different expansion points

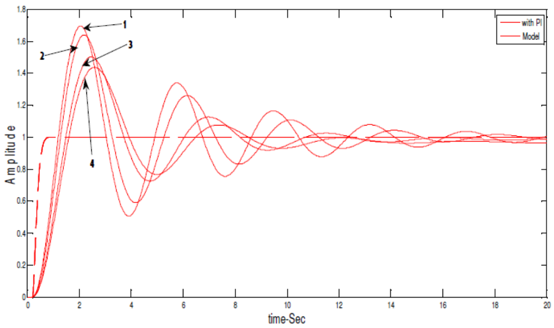
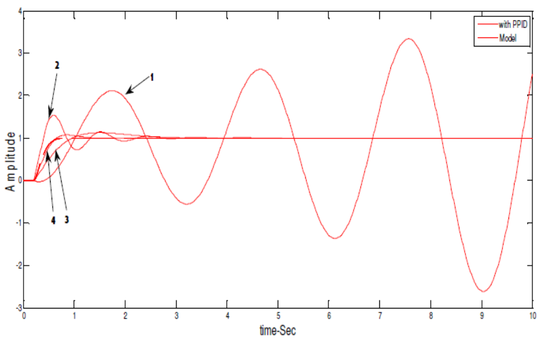


Figure 6.2: Step response of closed loop system with



PI controller for different expansion points

Figure 6.3: Step response of closed loop system with PPID controller for different expansion points

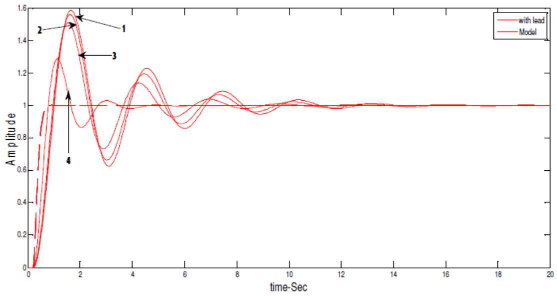


Figure 6.4: Step response of closed loop system with Lead/Lag controller for different expansion points

such that poles of the resulting closed loop system can be guaranteed to be in the left half of the s-plane. The number of expansion points, 89 depends upon the number of unknown parameters. In the present work, this problem of choosing the best expansion points has been cast as a constrained optimization problem which is solved by Genetic Algorithm (GA).

The optimization problem, in general can be stated as:

$$J = \int (Y_m(t) - Y_c(t))^2 \, dt$$

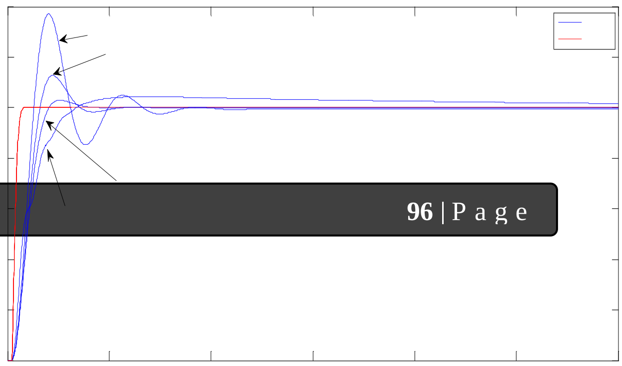
Where $y_m(t)$ is the response of the (desired) model, M(s) and $y_c(t)$ is the response of the closed loop system with designed controller C(s). The constraint ensures that the chosen expansions points will always yield a stable closed loop system.

Optimized expansion points from GA for AGTM/AGMP method

Table.2: Comparison of different controller performances using AGTM/AGMP method with optimized expansion points:

Type of controller	Expansion points(S)	Controller parameters			Stability (K)	Performance measure(J)
		K_p	K_i	K_d		
PID	[-1.4946 -0.9313 -1.36]	3.24	1.019	2.22	1	14.94
PI	[-0.1308 -0.3754]	0.539	-0.03	0	1	66.56
PPID	[0.0503 -1.2842 0.962]	0.8150	0.002	944.14	1	0.2171
Lead/Lag	[0.3848 -1.43 -69.53]	K	a	b	1	0.5379
		64.51	48.7	69.53		

The Step response of the closed loop system with PID with best expansion points obtained from Genetic Algorithm



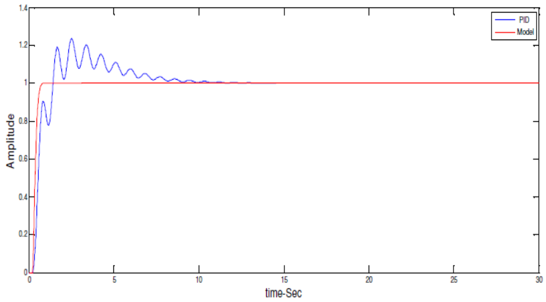


Figure 6.5: Step response of closed loop system with PID controller

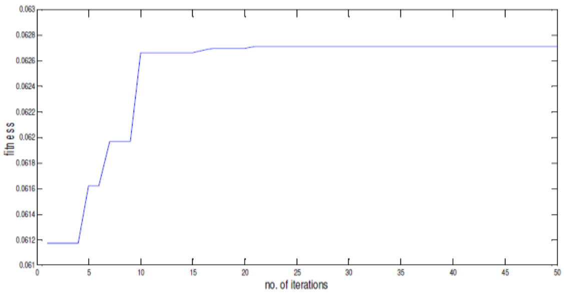


Figure 6.6: Variation of Fitness value with No. of generations for PID controller

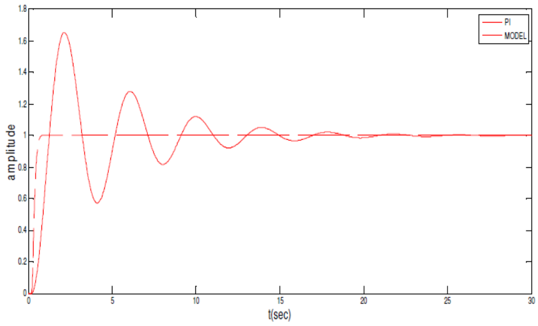


Figure 6.7: Step response of closed loop system with PI controller

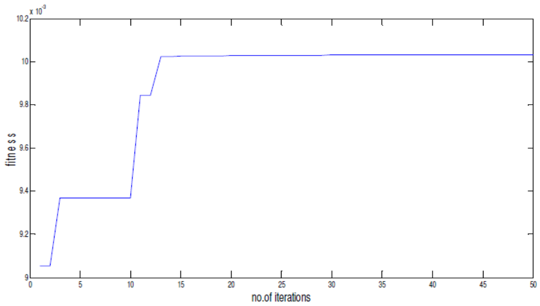


Figure 6.8: Variation of Fitness value with No. of generations for PI controller

The Step response of the closed loop system with PPID with best expansion points obtained from Genetic Algorithm

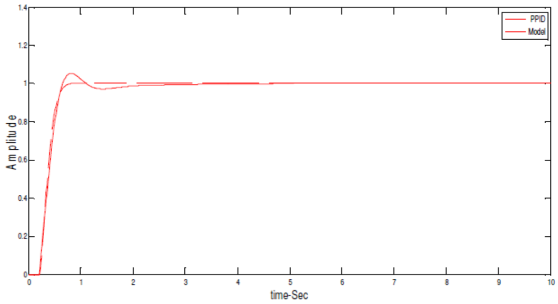


Figure 6.9: Step response of closed loop system with PPID controller

Optimal Pade Approximation Method[15]

Higher order controller K(s) is reduced to PID and PI controller structures

Design of PID controller

For expansion points, s= [1.82, 0.06, 1],

PID controller,

$$C_{PID}(s) = \frac{0.0785s^2+1.091s-0.005}{s}$$

Where K_D=0.0785; K_P=1.091; K_I=-0.005

- System is stable.
- Performance measure, J=18.66

Design of PI controller

For expansion points, s= [-5.42, -6.53],

- PI controller

$$C_{PI}(s) = \frac{1.044s+0.2629}{s}$$

Where, K_P=1.044,K_I=0.2629

- System is stable
- Performance measure, J=86.36

Design of PI controller

For expansion points, s= [-5.42, -6.53],

PI controller

$$C_{PI}(s) = \frac{1.044s+0.2629}{s}$$

Where, K_P=1.044,K_I =0.2629

System is stable Performance measure, J=86.36

Table 6.3: Comparison of different controller performances using Optimal Pade approximation method for randomly chosen expansion points:

Type of Controller	Expansion points	Controller Parameters			stability	Performance measure (J)
		K_d	K_p	K_i		
PID	[1.82 0.06 0.93]	-0.012	1.104	-0.006	Stable	21.6
	[1.82 0.06 1]	0.0785	1.091	-0.005	Stable	18.66
	[10 0.2 3]	-1.304	16.4	0.1378	unstable	--
PI	[-5.42 -6.53]		1.044	0.2629	stable	86.36
	[-3.92 -6.65]		1.063	0.1417	stable	73.16
	[1.92 -1.63]		1.01	0.5995	Unstable	--

Step responses of the closed-loop system with PID controller for the expansion points given in the Table 6.3:

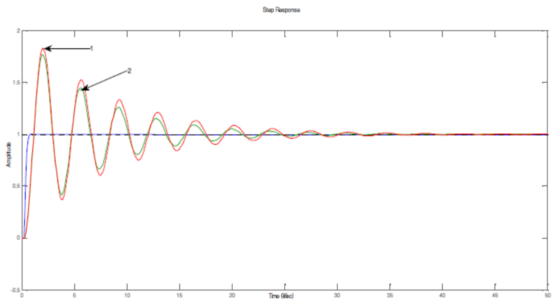


Figure 6.13: Step response of closed loop system with PID controller

Table.3: Comparison of different controller performances using OptimalPade approximation method with optimized expansion points:

Type of controller	Expansion points	Controller parameters			Performance measure(J)
		Kp	Ki	Kd	
PID	[2.15 0.94 -0.2]	-0.0054	0.018	1.09	15.04
PI	[-0.074 -0.47]	1.08	-0.033		67.38

The Step response of the closed loop system with PID with best expansion pointsobtained from Genetic Algorithm

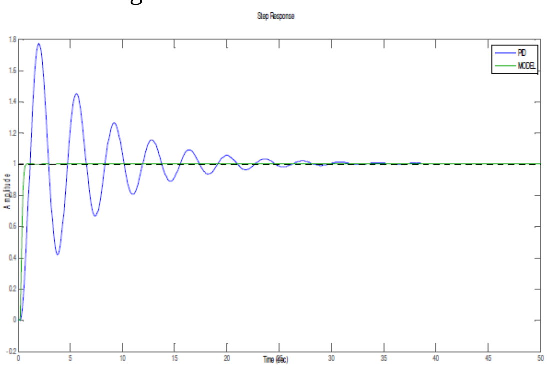


Figure 6.15: Step response of closed loop system with PID controller

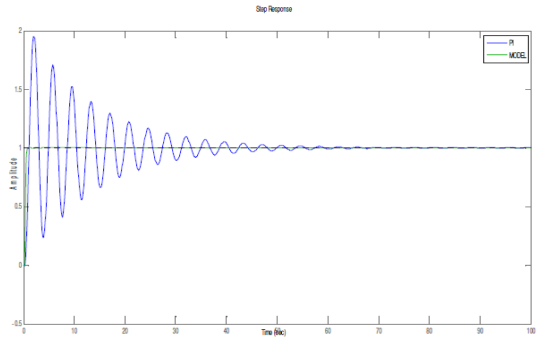


Figure 6.18: Variation of Fitness value with No. of generations for PI controller

Summary

Design of Model Matching Controller is done and further reduced to lower order controllers using AGTM/AGMP technique and Optimal Pade Approximation (OPA) method.
To assure the closed loop system stability and to reduce the error between the responses of the model and the closed loop system, the expansion points has to be optimized. GA optimization technique was used to achieve the optimal expansion points for different controllers.

VI. CONCLUSION AND FUTURE SCOPE

The controllers are designed for time delay systems based on the concept of model matching, model order reduction and GA optimization techniques. The algorithms developed using AGTM matching method and OPA results in linear algebraic equations whose solution leads to the controller parameters. The developed controller design methodologies are generated and are free from any random selection of expansion points
The significance of work is using the concept of model order reduction it leads to linear method of finding unknown parameters of controllers.

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